2018 Math Summer Assignment Name________________________________
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**Due September 7th, 2018**

Dear Incoming 7th Graders,

For your summer assignment there are two parts. The first past is a short reading assignment and part two is a math review on Delta Math. We will be using Delta Math throughout the coming school year, so it is a good idea to get acquainted with the site.

**Part I – Summer Reading Assignment**

"What is Mathematics?" "Who Invented Zero" and "What is Algebra" are the three articles you will be reading.

Our first unit will be based on these three articles. Also, we will discuss the concept of infinity *(and beyond…!)*

Please read the articles and complete the attached KWL graphic organizer for each article.

**Know** (to be completed before reading the article)

-- What do you know about mathematics and its definition?
-- What is zero and where did it come from?
-- What does Algebra mean to you?
-- What is infinity?

**Wonder** (To be completed before reading)

-- What do you wonder about mathematics?
-- What do you wonder about zero?
-- What do you wonder about Algebra?
-- What do you wonder about infinity?

**Learn** (To be completed after reading)

-- What did you learn about mathematics?
-- What did you learn about zero?
-- What did you learn about Algebra?
-- What did you learn about infinity?

**Part II – Summer Math Practice**

This part of the assignment will not be posted until August 1st. The idea of this assignment is to refresh you math skills before the school year starts, as well as give yourself and myself an idea of what you know and don't know going into 7th grade. There is a total of 8 assignments, I recommend working on two assignments per week starting the first week of August.

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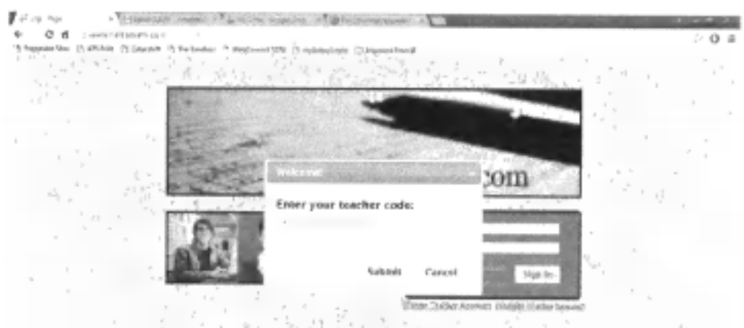
1. Go to www.deltamath.com




2. Click "Create New Account"

**3. Type in 960317 as your teacher code. Submit and Verify Teacher information**

Create Student Account ×

**Teacher Code:**

960317

Teacher Name: **Mr. Redmond**

Student and Login Information:

4. Set up your account using you CSS login information. Select "2018 Summer" for your class section. Click create account.

Student and Login Information:

**Section:** Summer 2017 ▾

**First Name:**

**Last Name:**

**Email:** brandonredmond@columbiasecondary.org

**Email (verify):**

**Password:** ••••••••••

**Password (verify):**

Create Account ✔ Cancel

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## Know—Wonder—Learn

**You will complete one KWL sheet for each article.**

Use this as an opportunity to reflect on what you know about mathematics already.

What is mathematics, *really*?
What actually *is* zero?
What is infinity?
Are there different levels of infinity? Larger? Smaller?

### Article 1 – What is mathematics?

| Know | Wonder | Learn |
| --- | --- | --- |
| What do you know about fractions? | What do you want to know? | What did you learn from the reading? |
| Complete before reading | Complete before reading | Complete after reading |
| | | |

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**Article 2 – Who Invented Zero**

| Know | Wonder | Learn |
|---|---|---|
| What do you know about fractions? | What do you want to know? | What did you learn from the reading? |
| Complete before reading | Complete before reading | Complete after reading |
| | | |

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**Article 3 –What is Algebra**

| Know | Wonder | Learn |
|---|---|---|
| What do you know about fractions? | What do you want to know? | What did you learn from the reading? |
| Complete before reading | Complete before reading | Complete after reading |
| | | |

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**Article 4 – No End in Sight: Debating the Existence of Infinity**

| Know | Wonder | Learn |
|---|---|---|
| What do you know about fractions? | What do you want to know? | What did you learn from the reading? |
| Complete before reading | Complete before reading | Complete after reading |
| | | |

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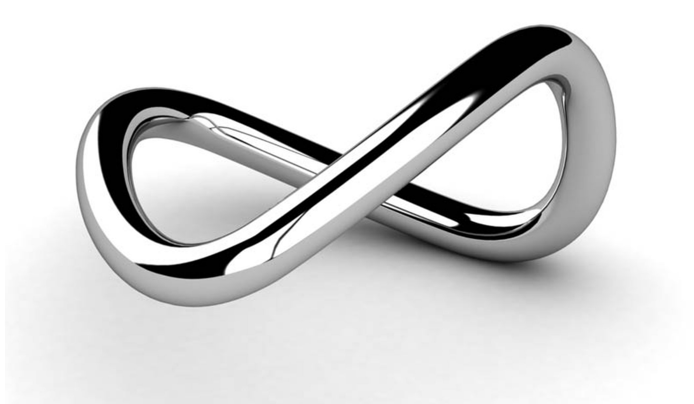
## Article 4

# No End in Sight: Debating the Existence of Infinity

By Denise Chow, Sci-Tech Editor



NEW YORK — Despite being in existence for more than 2,000 years, the concept of infinity has endured as an enigmatic, and oftentimes challenging, idea for mathematicians, physicists and philosophers. Does infinity really exist, or is it just part of the fabric of our imaginations?
A panel of scientists and mathematicians gathered to discuss some of the profound questions and controversies surrounding the concept of infinityhere Friday (May 31), as part of the World Science Festival, an annual celebration and exploration of science.
Part of the difficulty in trying to solve some of the abstract questions related to infinity is that these problems fall beyond the more established mathematical theories, said William Hugh Woodin, a mathematician at the University of California, Berkeley.

"It's kind of like mathematics lives on a stable island — we've built them a solid foundation," Woodin said. "Then, there's the wild land out there. That's infinity."

**Where it all began**

A philosopher named Zeno of Elea, who lived from 490 B.C. to 430 B.C, is credited with introducing the idea of infinity.

The concept was studied by ancient philosophers, including Aristotle, who questioned whether infinites could exist in a seemingly finite physical world, said Philip Clayton, dean of the Claremont School of Theology at Claremont Lincoln University in Claremont, Calif. Theologians, including Thomas Aquinas, used the infinite to explain the relationship between humans, God and the natural world.

In the 1870s, a German mathematician named Georg Cantor pioneered work in a field that became known as set theory. According to set theory, integers, which are numbers without a fraction or decimal component (such as 1, 5, -4), make up an infinite set that is countable. On the other hand, real numbers, which include integers, fractions and so-called irrational numbers, such as the square root of 2, are part of an infinite set that is uncountable.

This led Cantor to wonder about different types of infinity.

"If there are now two kinds of infinity — the countable kind and this continuous kind, which is bigger — are there other infinities? Is there some infinity that's sandwiched in between them?" said Steven Strogatz, a mathematician at Cornell University in Ithaca, N.Y.

Cantor believed that no infinities exist between the sets of integers and real numbers, but he was never able to prove it. His statement, however, became known as the continuum hypothesis, and mathematicians who tackled the problem in Cantor's footsteps were labeled set theorists.

**Exploring beyond**

Woodin is a set theorist, and has spent his life trying to solve the continuum hypothesis. To date, mathematicians have not been able to prove or disprove Cantor's postulation. Part of the problem is that the idea that there are more than two types of infinity is so abstract, Woodin said.

"There's no satellite you can build to go out and measure the continuum hypothesis," he explained. "There's nothing in our world around us that will help us determine whether or not the continuum hypothesis is true or false, as far as we know." [5 Seriously Mind-Boggling Math Facts]

Trickier still is the fact that some mathematicians have dismissed the relevance of this type of mathematical work.

"These people in set theory strike us, even in math, as sort of strange," Strogatz joked. But, he said he understands the importance of the work being done by set theorists, because if the continuum hypothesis is proven false, it could uproot basic mathematical principles in the same way that contradicting number theory would wipe out the bases for math and physics.

"We know that they're doing really deep, important work, and in principle, it's foundational work," Strogatz explained. "They're shaking the foundations that we're all working on, up on the second and third floors. If they mess something up, it could tip us all over."

**The future of mathematics**

Still, despite all of the uncertainties, the work done by set theorists could have positive ripple effects that serve to strengthen the foundations of mathematics, Woodin said.

"By investigating infinity, and to the extent that we can be successful, I think we make the case for the consistency of arithmetic," he explained. "That's a bit of a fanatical statement, but if infinity doesn't lead to a contradiction, certainly the finite doesn't lead to a contradiction. So, maybe by exploring the outer reaches to see if there is a contradiction, you gain some security."

The paradoxes that characterize the concept of infinity are perhaps best explained with the number pi, Strogatz said. Pi, one of the most recognizable mathematical constants, represents the ratio of a circle's circumference to its diameter. Among its myriad applications, pi can be used to find the area of a circle.

"Pi is typical of real numbers … in that it has this infinite amount of unpredictable information in it, and at the same time, is so totally predictable," Strogatz said. "There's nothing more orderly than a circle, which pi embodies — it's the very symbol of order and perfection. So this coexistence of perfect predictability and order, with this tantalizing mystery of infinite enigma built into the same object, is part of the pleasure of our subject and, I suppose, of infinity itself."

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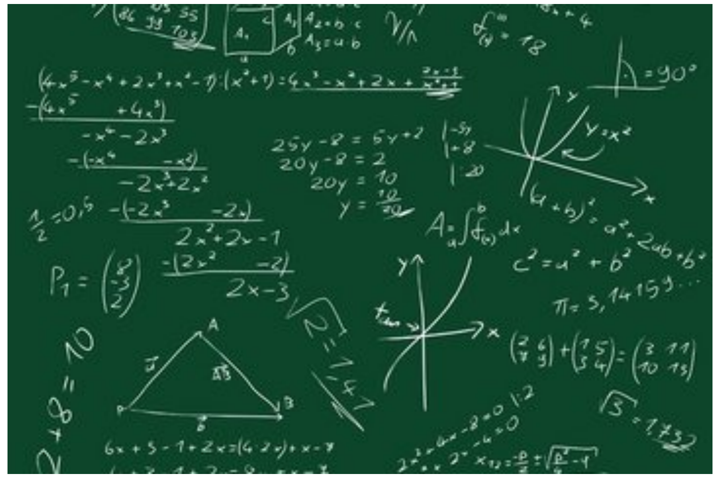
# Article 1



## What is Mathematics?

By Elaine J. Hom

Mathematics is the science that deals with the logic of shape, quantity and arrangement. Math is all around us, in everything we do. It is the building block for everything in our daily lives, including mobile devices, architecture (ancient and modern), art, money, engineering, and even sports.

Since the beginning of recorded history, mathematic discovery has been at the forefront of every civilized society, and in use in even the most primitive of cultures. The needs of math arose based on the wants of society. The more complex a society, the more complex the mathematical needs. Primitive tribes needed little more than the ability to count, but also relied on math to calculate the position of the sun and the physics of hunting.

### History of mathematics

Several civilizations — in China, India, Egypt, Central America and Mesopotamia — contributed to mathematics as we know it today. The Sumerians were the first people to develop a counting system. Mathematicians developed arithmetic, which includes basic operations, multiplication, fractions and square roots. The Sumerians' system passed through the Akkadian Empire to the Babylonians around 300 B.C. Six hundred years later, in America, the Mayans developed elaborate calendar systems and were skilled astronomers. About this time, the concept of zero was developed.

As civilizations developed, mathematicians began to work with geometry, which computes areas and volumes to make angular measurements and has many practical applications. Geometry is used in everything from home construction to fashion and interior design.

Geometry went hand in hand with algebra, invented in the ninth century by a Persian mathematician, Mohammed ibn-Musa al-Khowarizmi. He also developed quick methods for multiplying and diving numbers, which are known as algorithms — a corruption of his name.

Algebra offered civilizations a way to divide inheritances and allocate resources. The study of algebra meant mathematicians were solving linear equations and systems, as well as quadratics, and delving into positive and negative solutions. Mathematicians in ancient times also began to look at number theory. With origins in the construction of shape, number theory looks at figurate numbers, the characterization of numbers, and theorems.

**Math and the Greeks**

The study of math within early civilizations was the building blocks for the math of the Greeks, who developed the model of abstract mathematics through geometry. Greece, with its incredible architecture and complex system of government, was the model of mathematic achievement until modern times. Greek mathematicians were divided into several schools:

- **The Ionian School**, founded by Thales, who is often credited for having given the first deductive proofs and developing five basic theorems in plane geometry.
- **The Pythagorean School**, founded by Pythagoras, who studied proportion, plane and solid geometry, and number theory.
- **The Eleatic School**, which included Zeno of Elea, famous for his four paradoxes.
- **The Sophist School**, which is credited for offering higher education in the advanced Greek cities. Sophists provided instruction on public debate using abstract reasoning.
- **The Platonic School**, founded by Plato, who encouraged research in mathematics in a setting much like a modern university.
- **The School of Eudoxus**, founded by Eudoxus, who developed the theory of proportion and magnitude and produced many theorems in plane geometry
- **The School of Aristotle**, also known as the Lyceum, was founded by Aristotle and followed the Platonic school.

In addition to the Greek mathematicians listed above, a number of Greeks made an indelible mark on the history of mathematics. Archimedes, Apollonius, Diophantus, Pappus, and Euclid all came from this era. To better understand the sequence and how these mathematicians influenced each other, visit this timeline.

During this time, mathematicians began working with trigonometry. Computational in nature, trigonometry requires the measurement of angles and the computation of trigonometric functions, which include sine, cosine, tangent, and their reciprocals. Trigonometry relies on the synthetic geometry developed by Greek mathematicians like Euclid. For example, Ptolemy's theorem gives rules for the chords of the sum and difference of angles, which correspond to the sum and difference formulas for sines and cosines. In past cultures, trigonometry was applied to astronomy and the computation of angles in the celestial sphere.

After the fall of Rome, the development of mathematics was taken on by the Arabs, then the Europeans. Fibonacci was one of the first European mathematicians, and was famous for his theories on arithmetic, algebra, and geometry. The Renaissance led to advances that included decimal fractions, logarithms, and projective geometry. Number theory was greatly expanded upon, and theories like probability and analytic geometry ushered in a new age of mathematics, with calculus at the forefront.

**Development of calculus**

In the 17th century, Isaac Newton and Gottfried Leibniz independently developed the foundations for calculus. Calculus development went through three periods: anticipation, development and rigorization. In the anticipation stage, mathematicians were attempting to use techniques that involved infinite processes to find areas under curves or maximize certain qualities. In the development stage, Newton and Leibniz brought these techniques together through the derivative and integral. Though their methods were not always logically sound, mathematicians in the 18th century took on the rigorization stage, and were able to justify them and create the final stage of calculus. Today, we define the derivative and integral in terms of limits.

In contrast to calculus, which is a type of continuous mathematics, other mathematicians have taken a more theoretical approach. Discrete mathematics is the branch of math that deals with objects that can assume only distinct, separated value. Discrete objects can be characterized by integers, whereas continuous objects require real numbers. Discrete mathematics is the mathematical language of computer science, as it includes the study of algorithms. Fields of discrete mathematics include combinatorics, graph theory, and the theory of computation.

People often wonder what relevance mathematicians serve today. In a modern world, math such as applied mathematics is not only relevant, it's crucial. Applied mathematics is the branches of mathematics that are involved in the study of the physical, biological, or sociological world. The idea of applied math is to create a group of methods that solve problems in science. Modern areas of applied math include mathematical physics, mathematical biology, control theory, aerospace engineering, and math finance. Not only does applied math solve problems, but it also discovers new problems or develops new engineering disciplines. Applied mathematicians require expertise in many areas of math and science, physical intuition, common sense, and collaboration. The common approach in applied math is to build a mathematical model of a phenomenon, solve the model, and develop recommendations for performance improvement.

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While not necessarily an opposite to applied mathematics, pure mathematics is driven by abstract problems, rather than real world problems. Much of what's pursued by pure mathematicians can have their roots in concrete physical problems, but a deeper understanding of these phenomena brings about problems and technicalities. These abstract problems and technicalities are what pure mathematics attempts to solve, and these attempts have led to major discoveries for mankind, including the Universal Turing Machine, theorized by Alan Turing in 1937. The Universal Turing Machine, which began as an abstract idea, later laid the groundwork for the development of the modern computer. Pure mathematics is

abstract and based in theory, and is thus not constrained by the limitations of the physical world.

According to one pure mathematician, pure mathematicians prove theorems, and applied mathematicians construct theories. Pure and applied are not mutually exclusive, but they are rooted in different areas of math and problem solving. Though the complex math involved in pure and applied mathematics is beyond the understanding of most average Americans, the solutions developed from the processes have affected and improved the lives of all.

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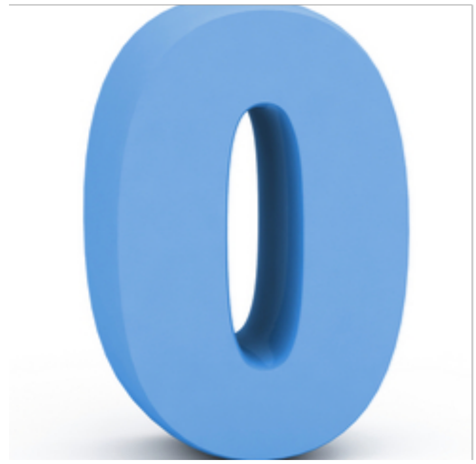
## Article 2



# Who Invented Zero?

Jessie Szalay, LiveScience Contributor

Though humans have always understood the concept of nothing or having nothing, the concept of zero is relatively new — it only fully developed in the fifth century A.D. Before then, mathematicians struggled to perform the simplest arithmetic calculations. Today, zero — both as a symbol (or numeral) and a concept meaning the absence of any quantity — allows us to perform calculus, do complicated equations, and to have invented computers.

**Early history: Angled wedges**

Zero was invented independently by the Babylonians, Mayans and Indians (although some researchers say the Indian number system was influenced by the Babylonians). The Babylonians got their number system from the Sumerians, the first people in the world to develop a counting system. Developed 4,000 to 5,000 years ago, the Sumerian system was positional — the value of a symbol depended on its position relative to other symbols. Robert Kaplan, author of "The Nothing That Is: A Natural History of Zero," suggests that an ancestor to the placeholder zero may have been a pair of angled wedges used to represent an empty number column. However, Charles Seife, author of "Zero: The Biography of a Dangerous Idea," disagrees that the wedges represented a placeholder.

The Sumerians' system passed through the Akkadian Empire to the Babylonians around 300 B.C. There, scholars agree, a symbol appeared that was clearly a placeholder — a way to tell 10 from 100 or to signify that in the number 2,025, there is no number in the hundreds column. Initially, the Babylonians left an empty space in their cuneiform number system, but when that became confusing, they added a symbol — double angled wedges — to represent the empty column. However, they never developed the idea of zero as a number.

**Zero in the Americas**

Six hundred years later and 12,000 miles from Babylon, the Mayans developed zero as a placeholder around A.D. 350 and used it to denote a placeholder in theirelaborate calendar systems. Despite being highly skilled mathematicians, the Mayans never used zero in equations, however. Kaplan describes the Mayan invention of zero as the "most striking example of the zero being devised wholly from scratch."

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**India: Where zero became a number**

Some scholars assert that the Babylonian concept wove its way down to India, but others give the Indians credit for developing zero independently.

The concept of zero first appeared in India around A.D. 458. Mathematical equations were spelled out or spoken in poetry or chants rather than symbols. Different words symbolized zero, or nothing, such as "void," "sky" or "space." In 628, a Hindu astronomer and mathematician named Brahmagupta developed a symbol for zero — a dot underneath numbers. He also developed mathematical operations using zero, wrote rules for reaching zero through addition and subtraction, and the results of using zero in equations. This was the first time in the world that zero was recognized as a number of its own, as both an idea and a symbol.

**From the Middle East to Wall Street**

Over the next few centuries, the concept of zero caught on in China and the Middle East. According to Nils-Bertil Wallin of YaleGlobal, by A.D. 773, zero reached Baghdad where it became part of the Arabic number system, which is based upon the Indian system.

A Persian mathematician, Mohammed ibn-Musa al-Khowarizmi, suggested that a little circle should be used in calculations if no number appeared in the tens place. The Arabs called this circle "sifr," or "empty." Zero was crucial to al-Khowarizmi, who used it to invent algebra in the ninth century. Al-Khowarizmi also developed quick methods for multiplying and dividing numbers, which are known as algorithms — a corruption of his name.

Zero found its way to Europe through the Moorish conquest of Spain and was further developed by Italian mathematician Fibonacci, who used it to do equations without an abacus, then the most prevalent tool for doing arithmetic. This development was highly popular among merchants, who used Fibonacci's equations involving zero to balance their books.

Wallin points out that the Italian government was suspicious of Arabic numbers and outlawed the use of zero. Merchants continued to use it illegally and secretively, and the Arabic word for zero, "sifr," brought about the word "cipher," which not only means a numeric character, but also came to mean "code."

By the 1600s, zero was used fairly widely throughout Europe. It was fundamental in Rene Descartes' Cartesian coordinate system and in Sir Isaac Newton's and Gottfried Wilhem Liebniz's developments of calculus. Calculus paved the way for physics, engineering, computers, and much of financial and economic theory.

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# What Is Algebra?

by Robert Coolman, Live Science Contributor

Algebra is a branch of mathematics dealing with symbols and the rules for manipulating those symbols. In elementary algebra, those symbols (today written as Latin and Greek letters) represent quantities without fixed values, known as variables. Just as sentences describe relationships between specific words, in algebra, equations describe relationships between variables. Take the following example:

I have two fields that total 1,800 square yards. Yields for each field are 2⁄3 gallon of grain per square yard and 1⁄2 gallon per square yard. The first field gave 500 more gallons than the second. What are the areas of each field?

It's a popular notion that such problems were invented to torment students, and this might not be far from the truth. This problem was almost certainly written to help students understand mathematics — but what's special about it is it's nearly 4,000 years old! According to Jacques Sesiano in "An Introduction to the History of Algebra" (AMS, 2009), this problem is based on a Babylonian clay tablet circa 1800 B.C. (VAT 8389, Museum of the Ancient Near East). Since these roots in ancient Mesopotamia, algebra has been central to many advances in science, technology, and civilization as a whole. The language of algebra has varied significantly across the history of all civilizations to inherit it (including our own). Today we write the problem like this:

$x + y = 1{,}800$ $\frac{2}{3}\cdot x - \frac{1}{2}\cdot y = 500$

The letters x and y represent the areas of the fields. The first equation is understood simply as "adding the two areas gives a total area of 1,800 square yards." The second equation is more subtle. Since x is the area of the first field, and the first field had a yield of two-thirds of a gallon per square yard, "2⁄3·x" — meaning "two-thirds times x" — represents the total amount of grain produced by the first field. Similarly "1⁄2·y" represents the total amount of grain produced by the second field. Since the first field gave 500 more gallons of grain than the second, the difference (hence, subtraction) between the first field's grain (2⁄3·x) and the second field's grain (1⁄2·y) is (=) 500 gallons.

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## Answer pops out

Of course, the power of algebra isn't in coding statements about the physical world. Computer scientist and author Mark Jason Dominus writes on his blog, The Universe of Discourse: "In the first phase you translate the problem into algebra, and then in the second phase you manipulate the symbols, almost mechanically, until the answer pops out as if by magic." While these manipulation rules derive from mathematical principles, the novelty and non-sequitur nature of "turning the crank" or "plugging and chugging" has been noticed by many students and professionals alike.

Here, we will solve this problem using techniques as they are taught today. And as a disclaimer, the reader does not need to understand each specific step to grasp the importance of this overall technique. It is my intention that the historical significance and the fact that we are able to solve the problem without any guesswork will inspire inexperienced readers to learn about these steps in greater detail. Here is the first equation again:

$$x + y = 1{,}800$$

We solve this equation for y by subtracting x from **each side of the equation**:

$$y = 1{,}800 - x$$

Now, we bring in the second equation:

$$\frac{2}{3} \cdot x - \frac{1}{2} \cdot y = 500$$

Since we found "1,800 – x" is equal to y, it may be **substituted** into the second equation:

$$\frac{2}{3} \cdot x - \frac{1}{2} \cdot (1{,}800 - x) = 500$$

Next, **distribute** the negative one-half (–1⁄2) across the expression "1,800 – x":

$$\frac{2}{3} \cdot x + (-\frac{1}{2} \cdot 1{,}800) + (-\frac{1}{2} \cdot -x) = 500$$

This **simplifies** to:

$$\frac{2}{3} \cdot x - 900 + \frac{1}{2} \cdot x = 500$$

Add the two fractions of x together and add 900 to **each side of the equation**:

$$\frac{7}{6} \cdot x = 1{,}400$$

Now, divide **each side of the equation** by 7/6:

$$x = 1{,}200$$

---

Thus, the first field has an area of 1,200 square yards. This value may be **substituted** into the first equation to determine y:

$$1{,}200 + y = 1{,}800$$

Subtract 1,200 from **each side of the equation** to solve for y:

$$y = 600$$

Thus, the second field has an area of 600 square yards.

Notice how often we employ the technique of doing an operation to **each side of an equation**. This practice is best understood as visualizing an equation as a scale with a known weight on one side and an unknown weight on the other. If we add or subtract the same amount of weight from each side, the scale remains balanced. Similarly, the scale remains balanced if we multiply or divide the weights equally.

While the technique of keeping equations balanced was almost certainly used by all civilizations to advance algebra, using it to solve this ancient Babylonian problem (as shown above) is anachronistic since this technique has only been central to algebra for the last 1,200 years.

**Before the Middle Ages**

Algebraic thinking underwent a substantial reform following the advancement by scholars of Islam's Golden Age. Until this point, the civilizations that inherited Babylonian mathematics practiced algebra in progressively elaborate "procedural methods." Sesiano further explains: A "student needed to memorize a small number of [mathematical] identities, and the art of solving these problems then consisted in transforming each problem into a standard form and calculating the solution." (As an aside, scholars from ancient Greece and India did practice symbolic language to learn about number theory.)

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An Indian mathematician and astronomer, Aryabhata (A.D. 476-550), wrote one of the earliest- known books on math and astronomy, called the "Aryabhatiya" by modern scholars. (Aryabhata did not title his work himself.) The work is "a small astronomical treatise written in 118 verses giving a summary of Hindu mathematics up to that time," according to

the University of St. Andrews, Scotland.

Here is a sample of Aryabhata's writing, in Sanskrit. This is verse 2.24, "Quantities from their difference and product":

Aryabhatiya, verse 2.24: "Quantities from their difference and product." Sanskrit, palm leaf, A.D. 499.

According to Kripa Shankar Shukla in "Aryabhatiya of Aryabhata" (Indian National Science Academy of New Delhi, 1976), this verse approximately translates to:

द्विकृतिगुणात्संवर्गात् द्विअन्तरवर्गेण संयुतात्मूलम् ।
अन्तरयुक्तं हीनं तद्गुणकार द्वयं दलितम् ।। २.२४ ।।

2.24: To determine two quantities from their difference and product, multiply the product by four, then add the square of the difference and take the square root. Write this result down in two slots. Increase the first slot by the difference and decrease the second by the difference. Cut each slot in half to obtain the values of the two quantities.

In modern algebraic notation, we write the difference and product like this:

The procedure is then written like this:

$$x - y = A\,(difference)\; x \cdot y = B\,(product)$$

$$x = [\sqrt{(4 \cdot B + A^2)} + A]/2\; y = [\sqrt{(4 \cdot B + A^2)} - A]/2$$

This is a variation of the quadratic formula. Similar procedures appear as far back as Babylonia, and represented the state of algebra (and its close ties to astronomy) for more than 3,500 years, across many civilizations: Assyrians, in the 10th century B.C.; Chaldeans, in the seventh century B.C.; Persians, in the sixth century B.C.; Greeks, in the fourth century B.C.; Romans, in the first century

A.D.; and Indians, in the fifth century A.D.

While such procedures almost certainly originated in geometry, it is important to note the original texts from each civilization say absolutely nothing about how such procedures *were determined*, and no efforts were made to *show proof* of their correctness. Written records addressing these problems first appeared in the Middle Ages.

**Algebra's adolescence**

The Golden Age of Islam, a period from the mid-seventh century to the mid-13th century, saw the spread of Greek and Indian mathematics to the Muslim world. In A.D. 820, Al-Khwārizmī, a faculty member of the House of Wisdom of Baghdad, published "Al-jabr wa'l muqabalah," or "The Compendious Book on Calculation by Completion and Balancing." It is from "al-jabr" that we derive our word "algebra." Al-Khwārizmī also developed quick methods for multiplying and dividing numbers, which are known as algorithms — a corruption of his name. He also suggested that a little circle should be used in calculations if no number appeared in the tens place — thus inventing the zero.

For the first time since its inception, the practice of algebra shifted its focus awayfrom *applying* procedural methods more toward means of *proving and deriving* such methods using geometry and the technique of doing operations to each side of an equation. According to Carl B. Boyer in "A History of Mathematics 3rd Ed." (2011, Wiley), Al-Khwārizmī found it "necessary that we should demonstrate geometrically the truth of the same problems which we have explained in numbers."

Medieval Muslim scholars wrote equations out as sentences in a tradition now knownas *rhetorical* algebra. Over the next 800 years, algebra progressed over a spectrum of rhetorical and symbolic language known as *syncopated* algebra. The pan-Eurasian heritage of knowledge that included mathematics, astronomy and navigation found its way to Europe between the 11th and 13th centuries, primarily through the Iberian Peninsula, which was known to the Arabs as Al-Andalus. Particular points of transmission to Europe were the 1085 conquest of Toledo by Spanish Christians, the 1091 re-claiming of Sicily by the Normans (after the Islamic conquest in 965) and the Crusader battles in the Levant from 1096 to 1303. Additionally, a number of Christian scholars such as Constantine the African (1017-1087), Adelard of Bath (1080-1152) and Leonardo Fibonacci (1170-1250) traveled to Muslim lands to learn sciences.

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**Maturation**

Fully symbolic algebra — as demonstrated at the beginning of the article — wouldn't be recognizable until the Scientific Revolution. René Descartes (1596-1650) used algebra we would recognize today in his 1637 publication "La Géométrie," which pioneered the practice of graphing algebraic equations. According to Leonard Mlodinow in "Euclid's Window" (Free Press, 2002), Descartes' "geometric methods were so crucial to his insights that he wrote that 'my entire physics is nothing other than geometry.'" Algebra, having departed from its procedural geometric partner 800 years earlier to develop into a symbolic language, had come full circle.